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The Pion Velocity at Chiral Restoration and the Vector Manifestation

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Abstract

We study the effects of Lorentz non-invariance on the physical pion velocity at the critical temperature T_c in an effective theory of hidden local symmetry (HLS) with the "vector manifestation" fixed point. We match at a "matching scale" Λ_M the axial-vector current correlator in the HLS with the one in the operator product expansion for QCD, and present the matching condition to determine the bare pion velocity. We find that the physical pion velocity, which is found to be one at $T = T_c$ when starting from the Lorentz invariant bare HLS, remains close to one with the Lorentz non-invariance, $v_\pi(T_c) = 0.83 - 0.99$. This result is quite similar to the pion velocity in dense matter.

1 Introduction

Recent developments on effective field theory based on hidden local symmetry (HLS) suggest a state-of-the-art scenario for the chiral symmetry restoration at a large N_f, high temperature and/or high density, see Ref. [1] for a review. In the "vector manifestation (VM)" of the HLS theory, the ρ mesons become massless and the longitudinal components of the ρ mesons and the pions form a chiral multiplet at the restoration point [2]. It has been shown that the vector manifestation is realized for large N_f [2], at the critical temperature [3] as well as at the critical density [4] for chiral symmetry restoration. That the vector meson mass vanishes at the critical temperature/density supports the in-medium scaling of the vector meson proposed by Brown and Rho, "BR scaling" [5], and has qualitatively important influences on the properties of hadrons in medium. It has been shown [6] that the HLS with the VM predicts – and not posits – that the vector susceptibility χ_V equals the axial-vector susceptibility χ_A as required by chiral invariance and that the pion velocity $v_\pi = 1$ with the pion decay constants $f_\pi^t \to 0$ and $f_\pi^s \to 0$ as $T \to T_c$. This behavior differs drastically from the scenario predicted by the standard chiral theory [8] where the degrees of freedom at chiral restoration relevant to the iso-vector susceptibility are the pions after integrating out the scalar field in the O(4) linear sigma model in the case of two-flavor QCD: The standard model prediction is that the pion velocity goes to zero at the chiral restoration point.

One of the most important ingredients to realize the VM is the Wilsonian matching [7, 1] obtained from the following general ansatz: Integrating out quark and gluon degrees of freedom at a matching scale Λ_M, we obtain the *bare* Lagrangian of the effective field theory (EFT). Then, we can determine the *bare* parameters of the bare Lagrangian by matching the EFT to the fundamental theory (QCD). Physical quantities are obtained by including quantum and thermal and/or dense effects through the renormalization group equations and thermal and/or dense loop. Since we integrate out the high energy modes, i.e., the quarks and gluons above Λ, in hot and/or dense matter, the bare parameters generically have the *intrinsic temperature and/or density dependence* [3, 4] which generally induces Lorentz symmetry violation in the bare EFT [6, 9].

In arriving at the HLS/VM results of Ref. [6], it was assumed that one can ignore Lorentz symmetry breaking in the bare HLS Lagrangian matched at a scale Λ_M to QCD in medium. In this paper, we lift that assumption. This is made possible by a "non-renormalization theorem" recently proven by one of the present authors (C.S.) [10]. The observation is based on the existence of a new fixed point in the VM, namely that, *the physical pion velocity does not receive any correction, either quantum or hadronic thermal, at the critical temperature.* This means that it suffices to compute the pion velocity at the level of bare HLS Lagrangian at the matching point to arrive at the *physical* pion velocity at the chiral transition. It is important to study the physical pion velocity since this quantity is a dynamical object, which controls the pion propagation in medium through a dispersion relation.

In the present work, we calculate the *bare* pion velocity by matching the axial-vector current correlators given by the HLS Lagrangian with Lorentz non-invariant terms taken into account, to the ones from the operator product expansion (OPE) in QCD. In Section 2, we briefly review HLS to define our notations and write down the HLS Lagrangian including the Lorentz symmetry violation. We present in Section 3 the Wilsonian matching condition to determine the bare pion velocity and its intrinsic temperature dependence in the low-temperature region. The bare pion velocity at the critical temperature is calculated in Section 4 by the Wilsonian

matching. Our results are summarized with a brief conclusion in Section 5.

2 HLS Theory Without Lorentz Invariance

The key observation of hidden local symmetry theory (HLS) is that any nonlinear sigma model defined in the coset space G/H is gauge-equivalent to a linear model possessing $G_{\rm global} \times H_{\rm local}$ symmetry. Here $G = \mathrm{SU}(N_f)_{\rm L} \times \mathrm{SU}(N_f)_{\rm R}$ is the global chiral symmetry and $H = \mathrm{SU}(N_f)_{\rm V}$ the HLS. In the HLS theory the basic ingredients are the gauge bosons $\rho_\mu = \rho_\mu^a T_a$ of the HLS and two $\mathrm{SU}(N_f)$-matrix valued variables $\xi_{\rm L}$ and $\xi_{\rm R}$. They are parameterized as $\xi_{\rm L,R} = e^{i\sigma/F_\sigma} e^{\mp i\pi/F_\pi}$, where $\pi = \pi^a T_a$ denote the pseudoscalar Nambu-Goldstone (NG) bosons associated with the spontaneous breaking of G and $\sigma = \sigma^a T_a$ #1 the NG bosons absorbed into the HLS gauge bosons ρ_μ which are identified with the vector mesons. F_π and F_σ are the relevant decay constants, and the parameter a is defined as $a \equiv F_\sigma^2/F_\pi^2$. $\xi_{\rm L}$ and $\xi_{\rm R}$ transform as $\xi_{\rm L,R}(x) \to h(x)\xi_{\rm L,R}(x) g_{\rm L,R}^\dagger$, where $h(x) \in H_{\rm local}$ and $g_{\rm L,R} \in G_{\rm global}$. The covariant derivatives of $\xi_{\rm L,R}$ are defined by $D_\mu \xi_{\rm L} = \partial_\mu \xi_{\rm L} - ig\rho_\mu \xi_{\rm L} + i\xi_{\rm L}\mathcal{L}_\mu$, and similarly with replacement $\mathrm{L} \leftrightarrow \mathrm{R}$, $\mathcal{L}_\mu \leftrightarrow \mathcal{R}_\mu$, where g is the HLS gauge coupling, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ denote the external gauge fields gauging the $G_{\rm global}$ symmetry. The HLS Lagrangian is given by

$$\mathcal{L} = F_\pi^2 \, \mathrm{tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\right] + F_\sigma^2 \, \mathrm{tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\right] + \mathcal{L}_{\rm kin}(\rho_\mu) \ , \tag{1}$$

where $\mathcal{L}_{\rm kin}(\rho_\mu)$ denotes the kinetic term of ρ_μ and

$$\hat{\alpha}^\mu_{\perp,\parallel} = \left(D_\mu \xi_{\rm R} \cdot \xi_{\rm R}^\dagger \mp D_\mu \xi_{\rm L} \cdot \xi_{\rm L}^\dagger \right) / (2i) \ . \tag{2}$$

As we have argued in the Introduction, the application of the Wilsonian matching in hot matter leads to the intrinsic temperature dependence of the bare parameters of the HLS Lagrangian which induces the Lorentz symmetry violation at the bare level [6, 9]. The Lorentz non-invariant HLS Lagrangian is constructed in Appendix A of Ref. [4]. At the lowest order, the Lagrangian is given by

$$\begin{aligned}
\tilde{\mathcal{L}} \ = \ & \left[(F^t_{\pi,\rm bare})^2 u_\mu u_\nu + F^t_{\pi,\rm bare} F^s_{\pi,\rm bare} \left(g_{\mu\nu} - u_\mu u_\nu \right) \right] \mathrm{tr}\left[\hat{\alpha}_\perp^\mu \hat{\alpha}_\perp^\nu \right] \\
& + \left[(F^t_{\sigma,\rm bare})^2 u_\mu u_\nu + F^t_{\sigma,\rm bare} F^s_{\sigma,\rm bare} \left(g_{\mu\nu} - u_\mu u_\nu \right) \right] \mathrm{tr}\left[\hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu \right] \\
& + \left[-\frac{1}{g^2_{L,\rm bare}} u_\mu u_\alpha g_{\nu\beta} - \frac{1}{2 g^2_{T,\rm bare}} \left(g_{\mu\alpha} g_{\nu\beta} - 2 u_\mu u_\alpha g_{\nu\beta} \right) \right] \mathrm{tr}\left[V^{\mu\nu} V^{\alpha\beta} \right] \ ,
\end{aligned} \tag{3}$$

where $F^t_{\pi,\rm bare}$ ($F^t_{\sigma,\rm bare}$) and $F^s_{\pi,\rm bare}$ ($F^s_{\sigma,\rm bare}$) denote the *bare* parameters associated with the temporal and spatial decay constants of the pion (of the σ). The unit four-vector needed to account for non-Lorentz-invariance in medium takes the value $u^\mu = (1, \mathbf{0})$ at the rest frame. Here it should be noticed that, due to the Lorentz symmetry violation, two variables $\xi_{\rm L}$ and $\xi_{\rm R}$ included in the 1-forms $\hat{\alpha}^\mu_\perp$ and $\hat{\alpha}^\mu_\parallel$ in Eq. (3) are parameterized as [10]

$$\xi_{\rm L,R} = e^{i\sigma/F^t_\sigma} e^{\mp i\pi/F^t_\pi} \ , \tag{4}$$

#1 These scalars are not to be confused with the scalar that figures in two-flavor linear sigma model.

where $F^t_{\pi,\mathrm{bare}}$ and $F^t_{\sigma,\mathrm{bare}}$ are the bare parameters associated with the temporal decay constants of the pion and the σ.

The terms of $\mathcal{O}(p^4)$ relevant to the present analysis are given by

$$\bar{\mathcal{L}}_{z_2} = \left[2z^L_{2,\mathrm{bare}} u_\mu u_\nu g_{\nu\beta} + z^T_{2,\mathrm{bare}} \left(g_{\mu\alpha} g_{\nu\beta} - 2u_\mu u_\alpha g_{\nu\beta}\right)\right] \mathrm{tr}\left[\hat{\mathcal{A}}^{\mu\nu}\hat{\mathcal{A}}^{\alpha\beta}\right] , \tag{5}$$

where the parameters $z^L_{2,\mathrm{bare}}$ and $z^T_{2,\mathrm{bare}}$ correspond in medium to the vacuum parameter $z_{2,\mathrm{bare}}$ [7, 1] at $T = \mu = 0$. $\hat{\mathcal{A}}^{\mu\nu}$ is defined by

$$\hat{\mathcal{A}}^{\mu\nu} = \frac{1}{2}\left[\xi_{\mathrm{R}}\mathcal{R}^{\mu\nu}\xi^\dagger_{\mathrm{R}} - \xi_{\mathrm{L}}\mathcal{L}^{\mu\nu}\xi^\dagger_{\mathrm{L}}\right] , \tag{6}$$

where $\mathcal{R}^{\mu\nu}$ and $\mathcal{L}^{\mu\nu}$ are the field-strength tensors of the external gauge fields $\mathcal{R}_\mu$ and $\mathcal{L}_\mu$:

$$\begin{aligned} \mathcal{R}^{\mu\nu} &= \partial^\mu \mathcal{R}^\nu - \partial^\nu \mathcal{R}^\mu - i\left[\mathcal{R}^\mu , \mathcal{R}^\nu\right] , \\ \mathcal{L}^{\mu\nu} &= \partial^\mu \mathcal{L}^\nu - \partial^\nu \mathcal{L}^\mu - i\left[\mathcal{L}^\mu , \mathcal{L}^\nu\right] . \end{aligned} \tag{7}$$

3 Matching Conditions for Bare Pion Velocity

In this section, we present the matching conditions to determine the bare pion velocity including the effect of Lorentz symmetry breaking at the bare level.

The Wilsonian matching is carried out by matching the vector and axial-vector current correlators derived from the HLS with those from the OPE in QCD. The axial-vector current correlator is defined by

$$G^{\mu\nu}_A(q_0 = i\omega_n, \vec{q}; T)\delta_{ab} = \int_0^{1/T} d\tau \int d^3\vec{x} e^{-i(\vec{q}\cdot\vec{x}+\omega_n\tau)} \left\langle J^\mu_{5a}(\tau,\vec{x}) J^\nu_{5b}(0,\vec{0})\right\rangle_\beta , \tag{8}$$

where J^μ_{5a} is the axial-vector current, $\omega_n = 2n\pi T$ is the Matsubara frequency, $(a,b) = 1, \ldots, N_f^2 - 1$ denotes the flavor index and $\langle\ \rangle_\beta$ the thermal average. The correlator for Minkowski momentum is obtained by the analytic continuation of q_0. It is convenient to decompose the correlator into

$$G^{\mu\nu}_A(q_0,\vec{q}) = q^2 P^{\mu\nu}_L G^L_A(q_0,\bar{q}) + q^2 P^{\mu\nu}_T G^T_A(q_0,\bar{q}) , \tag{9}$$

where we define $\bar{q} = |\vec{q}|$ and the polarization tensors are given by

$$\begin{aligned} P^{\mu\nu}_T &\equiv g^\mu_i \left(\delta_{ij} - \frac{q_i q_j}{\bar{q}^2}\right) g^\nu_j \\ &= \left(g^{\mu\alpha} - u^\mu u^\alpha\right)\left(-g_{\alpha\beta} - \frac{q^\alpha q^\beta}{\bar{q}^2}\right)\left(g^{\beta\nu} - u^\beta u^\nu\right) , \\ P^{\mu\nu}_L &\equiv -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - P^{\mu\nu}_T . \end{aligned} \tag{10}$$

The bare Lagrangian is determined at the matching scale Λ through the matching and is expressed in terms of the bare parameters as the sum of Eqs. (3) and (5). From this bare Lagrangian, the current correlator at the matching scale is constructed as follows [#2]:

$$G^L_{A(\mathrm{HLS})}(q_0,\bar{q}) = \frac{F^t_{\pi,\mathrm{bare}} F^s_{\pi,\mathrm{bare}}}{-[q_0^2 - V^2_{\pi,\mathrm{bare}}\bar{q}^2]} - 2z^L_{2,\mathrm{bare}} ,$$

[#2] The current correlator below Λ is obtained by including quantum and hadronic corrections through the renormalization group equations and thermal and/or dense loop.

$$G^T_{A(\mathrm{HLS})}(q_0,\bar{q}) = -\frac{F^t_{\pi,\mathrm{bare}}F^s_{\pi,\mathrm{bare}}}{q^2} - 2\frac{q_0^2 z^L_{2,\mathrm{bare}} - \bar{q}^2 z^T_{2,\mathrm{bare}}}{q^2} \ , \tag{11}$$

where $V_{\pi,\mathrm{bare}} = F^s_{\pi,\mathrm{bare}}/F^t_{\pi,\mathrm{bare}}$ is the bare pion velocity. To perform the matching, we regard $G_A^{L,T}$ as functions of $-q^2$ and $\bar{q}^2$ instead of q_0 and $\bar{q}$, and expand $G_A^{L,T}$ in a Taylor series around $\bar{q} = |\vec{q}| = 0$ in $\bar{q}^2/(-q^2)$ as follows:

$$\begin{aligned} G^L_A(-q^2,\bar{q}^2) &= G_A^{L(0)}(-q^2) + G_A^{L(1)}(-q^2)\bar{q}^2 + \cdots, \\ G^T_A(-q^2,\bar{q}^2) &= G_A^{T(0)}(-q^2) + G_A^{T(1)}(-q^2)\bar{q}^2 + \cdots. \end{aligned} \tag{12}$$

In the following, we determine the bare pion velocity $V_{\pi,\mathrm{bare}}$ from $G_A^{L(0)}$ and $G_A^{L(1)}$ via the matching.

Expanding $G_A^{(\mathrm{HLS})L}$ in Eq. (11) in terms of $\bar{q}^2/(-q^2)$, we obtain

$$G_A^{(\mathrm{HLS})L(0)}(-q^2) = \frac{F^t_{\pi,\mathrm{bare}}F^s_{\pi,\mathrm{bare}}}{-q^2} - 2z^L_{2,\mathrm{bare}} \ , \tag{13}$$

$$G_A^{(\mathrm{HLS})L(1)}(-q^2) = \frac{F^t_{\pi,\mathrm{bare}}F^s_{\pi,\mathrm{bare}}(1-V^2_{\pi,\mathrm{bare}})}{(-q^2)^2}. \tag{14}$$

On the other hand, the correlator $G_A^{\mu\nu}$ in the QCD sector to be given in OPE is more involved. Our strategy goes as follows. Since the effect of Lorentz non-invariance in medium has been more extensively studied in dense matter, we first examine the form of the relevant correlator in dense matter following Refs. [11, 12]. The current correlator $\tilde{G}^{\mu\nu}$ constructed from the current defined by

$$J^{(q)}_\mu = \bar{q}\gamma_\mu q, \quad \text{or} \quad J^{(q)}_{5\mu} = \bar{q}\gamma_5\gamma_\mu q, \tag{15}$$

is given by

$$\begin{aligned} \tilde{G}^{\mu\nu}(q_0,\bar{q}) &= (q^\mu q^\nu - g^{\mu\nu}q^2)\left[-c_0 \ln|Q^2| + \sum_n \frac{1}{Q^n}A^{n,n}\right] \\ &+ \sum_{\tau=2}\sum_{k=1}[-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2] \\ &\quad \times q^{\mu_3}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k+\tau-2}}A^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}} \\ &+ \sum_{\tau=2}\sum_{k=1}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]q^{\mu_1}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k+\tau-2}}C^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}, \end{aligned} \tag{16}$$

where $Q^2 = -q^2$. $\tau = d - s$ denotes the twist, and $s = 2k$ is the number of spin indices of the operator of dimension d. Here $A^{n,n}$ represents the contribution from the Lorentz invariant operators such as $A^{4,4} = \frac{1}{6}\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_\rho$. $A^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}$ and $C^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}$ are the residual Wilson coefficient times matrix element of dimension d and twist τ; e.g., $A^{6,2}_{\mu_1\mu_2\mu_3\mu_4}$ is given by

$$A^{6,2}_{\mu_1\mu_2\mu_3\mu_4} = i\left\langle \mathcal{ST}\left(\bar{q}\gamma_{\mu_1}D_{\mu_2}D_{\mu_3}D_{\mu_4}q\right)\right\rangle_\rho \ , \tag{17}$$

where we have introduced the symbol $\mathcal{ST}$ which makes the operators symmetric and traceless with respect to the Lorentz indices. The general tensor structure of the matrix element of

$A^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}$ is given in Ref. [14]. For $k=2$, it takes the following form:

$$A_{\alpha\beta\lambda\sigma} = \Big[p_\alpha p_\beta p_\lambda p_\sigma - \frac{p^2}{8}(p_\alpha p_\beta g_{\lambda\sigma} + p_\alpha p_\lambda g_{\beta\sigma} + p_\alpha p_\sigma g_{\lambda\beta} + p_\beta p_\lambda g_{\alpha\sigma} + p_\beta p_\sigma g_{\alpha\lambda} + p_\lambda p_\sigma g_{\alpha\beta}) + \frac{p^4}{48}(g_{\alpha\beta}g_{\lambda\sigma} + g_{\alpha\lambda}g_{\beta\sigma} + g_{\alpha\sigma}g_{\beta\lambda})\Big]A_4 \tag{18}$$

For $\tau=2$ with arbitrary k, we have [11]:

$$\begin{aligned} A^{2k+2,2}_{2k} &= C^q_{2,2k}A^q_{2k} + C^{\rm G}_{2,2k}A^{\rm G}_{2k} \\ C^{2k+2,2}_{2k} &= C^q_{L,2k}A^q_{2k} + C^{\rm G}_{L,2k}A^{\rm G}_{2k}\ , \end{aligned} \tag{19}$$

where $C^q_{2,2k} = 1+\mathcal{O}(\alpha_s)$, $C^{q,{\rm G}}_{L,2k} \sim \mathcal{O}(\alpha_s)$ and $C^{\rm G}_{2,2k} \sim \mathcal{O}(\alpha_s)$ (with the superscripts q and G standing respectively for quark and gluon) are the Wilson coefficients in the OPE [11]. The quantities A^q_n and $A^{\rm G}_n$ are defined by

$$\begin{aligned} A^q_n(\mu) &= 2\int_0^1 dx\ x^{n-1}[q(x,\mu)+\bar{q}(x,\mu)] \\ A^{\rm G}_n(\mu) &= 2\int_0^1 dx\ x^{n-1}G(x,\mu)\ , \end{aligned} \tag{20}$$

where $q(x,\mu)$ and $G(x,\mu)$ are quark and gluon distribution functions respectively. We observe that (16) consists of three classes of terms: One is independent of the background, i.e., density in this case, the second consists of scalar operators with various condensates $\langle\mathcal{O}\rangle_\rho$ and the third is made up of non-scalar operators whose matrix elements in dense matter could not be simply expressed in terms of various condensates $\langle\mathcal{O}\rangle_\rho$.

It is clear that Eq. (16) is a general expression that can be applied equally well to heat-bath systems. Thus we can simply transcribe (16) to the temperature case by replacing the condensates $\langle\mathcal{O}\rangle_\rho$ by $\langle\mathcal{O}\rangle_T$ and the quantities $A^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}$ and $C^{2k+\tau,\tau}_{\mu_1\cdots\mu_{2k}}$ by the corresponding quantities in heat bath. The higher the twist of operators becomes, the more these operators are suppressed since the dimensions of such operators become higher and the power of $1/Q^2$ appear. Thus in the following, we restrict ourselves to contributions from the twist 2 ($\tau=2$) operators. Then the temperature-dependent correlator can be written as

$$\begin{aligned} G^{\mu\nu}_A(q_0,\bar{q}) = (q^\mu q^\nu - g^{\mu\nu}q^2)\frac{-1}{4}\Bigg[&\frac{1}{2\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{6Q^4}\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_T \\ &- \frac{2\pi\alpha_s}{Q^6}\Big\langle\Big(\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d\Big)^2\Big\rangle_T \\ &- \frac{4\pi\alpha_s}{9Q^6}\Big\langle\Big(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\Big)\sum_q^{u,d,s}\bar{q}\gamma^\mu\lambda^a q\Big\rangle_T\Bigg] \\ + [-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} &+ q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2] \\ \times [\frac{4}{Q^4}A^{4,2}_{\mu_1\mu_2} &+ \frac{16}{Q^8}q^{\mu_3}q^{\mu_4}A^{6,2}_{\mu_1\mu_2\mu_3\mu_4}]\ , \end{aligned} \tag{21}$$

where $G^{\mu\nu}_A$ is constructed from the axial-vector current associated with the iso-triplet channel defined by

$$J_{5\mu} = \frac{1}{2}(\bar{u}\gamma_5\gamma_\mu u - \bar{d}\gamma_5\gamma_\mu d), \tag{22}$$

and we keep terms only up to the order of $1/Q^8$ for $A^{2k+2,2}_{\mu_1\cdots\mu_{2k}}$. The λ^a denote the $SU(3)$ color matrices normalized as $\mathrm{tr}[\lambda^a\lambda^b] = 2\delta^{ab}$. Here we have dropped the terms with $C^{2k+2,2}_{\mu_1\cdots\mu_{2k}}$ in the non-scalar operators since they are of higher order in both $1/(Q^2)^n$ and α_s compared to the terms in the first line of Eq. (21). The temperature dependence of $A^{4,2}_{\mu_1\mu_2}$ and $A^{6,2}_{\mu_1\mu_2\mu_3\mu_4}$, implicit in Eq. (21), will be specified below.

In order to effectuate the Wilsonian matching, we should in principle evaluate the condensates and temperature-dependent matrix elements of the non-scalar operators in Eq.(21) at the given scale Λ_M and temperature T in terms of QCD variables only. This can presumably be done on lattice. However no complete information is as yet available from model-independent QCD calculations. We are therefore compelled to resort to indirect methods and we adopt here an approach borrowed from QCD sum-rule calculations.

Let us first evaluate the quantities that figure in Eq.(21) at low temperature. In low temperature regime, only the pions are expected to be thermally excited. In the dilute pion-gas approximation, $\langle \mathcal{O}\rangle_T$ is evaluated as

$$\langle \mathcal{O}\rangle_T \simeq \langle \mathcal{O}\rangle_0 + \sum_{a=1}^{3} \int \frac{d^3p}{2\epsilon(2\pi)^3} \langle \pi^a(\vec{p})|\mathcal{O}|\pi^a(\vec{p})\rangle n_B(\epsilon/T), \tag{23}$$

where $\epsilon = \sqrt{\bar{p}^2 + m_\pi^2}$ and n_B is the Bose-Einstein distribution. As an example, we consider the operator of $(\tau, s) = (2, 4)$ that contributes to both $G_A^{L(0)}$ and $G_A^{L(1)}$. Noting that $G_A^L(q_0, \bar{q}) = G_{A00}/\bar{q}^2$, we evaluate $G_{A00}(q_0, \bar{q})$.

$$\begin{aligned} G^{(\tau=2,s=4)}_{\mathrm{A00}}(q_0,\bar{q}) \;&=\; \frac{3}{4}\int \frac{d^3p}{2\epsilon(2\pi)^3}\frac{16}{Q^8}[-q^\alpha q^\beta + g^{0\alpha}q^0q^\beta + q^0q^\alpha g^{0\beta} + g^{0\alpha}g^{0\beta}Q^2] \\ &\quad \times q^\lambda q^\sigma A^{6,2(\pi)}_{\alpha\beta\lambda\sigma} n_B(\epsilon/T), \end{aligned} \tag{24}$$

where $A^{6,2(\pi)}_{\alpha\beta\lambda\sigma}$ is given by [#3]

$$\begin{aligned} A^{6,2(\pi)}_{\alpha\beta\lambda\sigma} = \Big[& p_\alpha p_\beta p_\lambda p_\sigma - \frac{p^2}{8}(p_\alpha p_\beta g_{\lambda\sigma} + p_\alpha p_\lambda g_{\beta\sigma} + p_\alpha p_\sigma g_{\lambda\beta} + p_\beta p_\lambda g_{\alpha\sigma} \\ & + p_\beta p_\sigma g_{\alpha\lambda} + p_\lambda p_\sigma g_{\alpha\beta}) + \frac{p^4}{48}(g_{\alpha\beta}g_{\lambda\sigma} + g_{\alpha\lambda}g_{\beta\sigma} + g_{\alpha\sigma}g_{\beta\lambda})\Big] A_4^\pi \ , \end{aligned} \tag{25}$$

where A_4^π carries the temperature dependence. Taking $m_\pi^2 = 0$, we see that the terms with p^2 and p^4 in Eq. (25) are zero.

From Eqs. (21), (23) and (24), we obtain

$$\begin{aligned} G_A^{(\mathrm{OPE})L(0)}(-q^2) \;&=\; \frac{-1}{3} g^{\mu\nu} G^{(\mathrm{OPE})(0)}_{A,\mu\nu} \\ &=\; \frac{-1}{4}\left[\frac{1}{2\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{6Q^4}\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_T \right. \\ &\qquad - \frac{2\pi\alpha_s}{Q^6}\left\langle\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d\right)^2\right\rangle_T \end{aligned}$$

[#3] For the general tensor structure of the matrix elements with a polarized spin-one target, say, along the beam direction in scattering process, see Ref. [16].

$$
\begin{aligned}
& - \frac{4\pi\alpha_s}{9Q^6}\Big\langle \Big(\bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d \Big) \sum_q^{u,d,s} \bar{q}\gamma^\mu \lambda^a q \Big\rangle_T \Bigg] \\
& + \frac{\pi^2}{30}\frac{T^4}{Q^4} A_2^{\pi(u+d)} - \frac{16\pi^4}{63}\frac{T^6}{Q^6} A_4^{\pi(u+d)} . \qquad (26)
\end{aligned}
$$

$G_A^{(\mathrm{OPE})L(1)}$ takes the following form

$$
G_A^{(\mathrm{OPE})L(1)} = \frac{32}{105}\pi^4 \frac{T^6}{Q^8} A_4^{\pi(u+d)} . \qquad (27)
$$

We now proceed to estimate the pion velocity by matching to QCD.

First we consider the matching between $G_A^{(\mathrm{HLS})L(0)}$ and $G_A^{(\mathrm{OPE})L(0)}$. From Eqs. (13) and (26), we obtain

$$
\begin{aligned}
(-q^2)\frac{d}{d(-q^2)} G_A^{(\mathrm{HLS})L(0)} &= -\frac{F^t_{\pi,\mathrm{bare}} F^s_{\pi,\mathrm{bare}}}{Q^2} , \\
(-q^2)\frac{d}{d(-q^2)} G_A^{(\mathrm{OPE})L(0)} &= \frac{-1}{8\pi^2}\left[\left(1 + \frac{\alpha_s}{\pi}\right) + \frac{2\pi^2}{3}\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle_T}{Q^4} + \pi^3 \frac{1408}{27}\frac{\alpha_s \langle \bar{q}q \rangle_T^2}{Q^6} \right] \\
& \quad - \frac{\pi^2}{15}\frac{T^4}{Q^4} A_2^{\pi(u+d)} + \frac{16\pi^4}{21}\frac{T^6}{Q^6} A_4^{\pi(u+d)} . \qquad (28)
\end{aligned}
$$

Matching them at $Q^2 = \Lambda_M^2$ [#4], we obtain

$$
\begin{aligned}
\frac{F^t_{\pi,\mathrm{bare}} F^s_{\pi,\mathrm{bare}}}{\Lambda_M^2} &= \frac{1}{8\pi^2}\left[\left(1 + \frac{\alpha_s}{\pi}\right) + \frac{2\pi^2}{3}\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle_T}{\Lambda_M^4} + \pi^3 \frac{1408}{27}\frac{\alpha_s \langle \bar{q}q \rangle_T^2}{\Lambda_M^6} \right] \\
& \quad + \frac{\pi^2}{15}\frac{T^4}{\Lambda_M^4} A_2^{\pi(u+d)} - \frac{16\pi^4}{21}\frac{T^6}{\Lambda_M^6} A_4^{\pi(u+d)} \\
&\equiv G_0 . \qquad (29)
\end{aligned}
$$

Next we consider the matching between $G_A^{(\mathrm{HLS})L(1)}$ and $G_A^{(\mathrm{OPE})L(1)}$. From Eqs. (14) and (27), we have

$$
\frac{F^t_{\pi,\mathrm{bare}} F^s_{\pi,\mathrm{bare}} (1 - V^2_{\pi,\mathrm{bare}})}{\Lambda_M^2} = \frac{32}{105}\pi^4 \frac{T^6}{\Lambda_M^6} A_4^{\pi(u+d)} . \qquad (30)
$$

Noting that the right-hand-side of this expression is positive, we verify that

$$
V_{\pi,\mathrm{bare}} < 1 \qquad (31)
$$

which is consistent with the causality.

[#4] The current correlator is expanded in positive power of Q^2 in the HLS theory, while in negative power of Q^2 in the OPE. Thus we cannot match the whole Q^2-dependence between the HLS theory and OPE. However if there exists an overlapping area around a scale Λ, we can require the matching condition at that Λ. In fact, the Wilsonian matching at $T = 0$ in three flavor QCD was shown to give several predictions in remarkable agreement with experiments [7, 1]. This strongly suggests that there exists such an overlapping region. As discussed in Ref. [1], $\Lambda_M \ll \Lambda_{\mathrm{HLS}}$ can be justified in the large N_c limit, where Λ_{HLS} denotes the scale at which the HLS theory breaks down. We obtain the matching conditions in $N_c = 3$ by extrapolating the conditions in large N_c. As we mentioned above, the success of the Wilsonian matching at $T = 0$ with taking the matching scale as $\Lambda_M = 1.1$ GeV shows that this extrapolation is valid.

The bare pion velocity can be obtained by dividing Eq. (30) with Eq. (29). What we obtain is the deviation from the speed of light:

$$\delta_{\rm bare} \equiv 1 - V_{\pi,\rm bare}^2 = \frac{1}{G_0}\frac{32}{105}\pi^4\frac{T^6}{\Lambda_M^6}A_4^{\pi(u+d)}. \tag{32}$$

This should be valid at low temperature. We note that the Lorentz non-invariance does not appear when we consider the operator with $s = 2$, and that the operator with $s = 4$ generates the Lorentz non-invariance. This is consistent with the fact that G_A^L including up to the operator with $s = 2$ is expressed as the function of only Q^2 [15]. Equation (32) implies that the intrinsic temperature dependence starts from the $\mathcal{O}(T^6)$ contribution. On the other hand, the hadronic thermal correction to the pion velocity starts from the $\mathcal{O}(T^4)$ [9]. [There are $\mathcal{O}(T^2)$ corrections to $[f_\pi^t]^2$ and $[f_\pi^t f_\pi^s]$, but they are canceled with each other in the pion velocity.] Thus the hadronic thermal effect is dominant in low temperature region. At the critical temperature, the $\mathcal{O}(T^2)$ corrections to the pion velocity are also cancelled as in the low-temperature region. Furthermore there are no $\mathcal{O}(T^4)$ corrections to either $[f_\pi^t]^2$ or $[f_\pi^t f_\pi^s]$. Thus hadronic thermal corrections to the pion velocity are absent due to the protection by the VM. Therefore there remains only the intrinsic temperature dependence determined via the Wilsonian matching [10].

4 The Pion Velocity at Critical Temperature

In this section, we wish to evaluate the physical pion velocity at the critical temperature T_c starting from the Lorentz non-invariant bare Lagrangian. According to the non-renormalization theorem [10], the *bare* velocity so calculated should correspond to the *physical* pion velocity *at* the chiral transition point. Now, in the VM, bare parameters are determined by matching the HLS to QCD at the matching scale Λ_M and at temperature $T = T_c$.

We begin with a summary of the pion velocity found in the HLS/VM theory with Lorentz invariance [6, 10, 9]. The pion velocity is given by [6][#5]

$$v_\pi^2(\bar{q}) \quad = \quad \frac{F_\pi^2(0) + {\rm Re}\,\bar{\Pi}_\perp^s(\bar{q},\bar{q};T)}{F_\pi^2(0) + {\rm Re}\,\bar{\Pi}_\perp^t(\bar{q},\bar{q};T)} \ , \tag{33}$$

where $\bar{\Pi}_\perp^{s,t}(\bar{q},\bar{q};T)$ is the axial-vector two-point function [6] that represents hadronic thermal corrections. The two-point function $\bar{\Pi}_\perp^{\mu\nu}$ is decomposed into four components,

$$\bar{\Pi}_\perp^{\mu\nu} = u^\mu u^\nu \bar{\Pi}_\perp^t + (g^{\mu\nu} - u^\mu u^\nu)\bar{\Pi}_\perp^s + P_L^{\mu\nu}\bar{\Pi}_\perp^L + P_T^{\mu\nu}\bar{\Pi}_\perp^T \ . \tag{34}$$

The VM dictates that if one ignores Lorentz non-invariance in the bare Lagrangian in medium, the pion velocity approaches the speed of light as $T \to T_c$ [6, 9].

In the following, we extend the matching condition valid at low temperature, Eq. (32), to near the critical temperature, and determine the bare pion velocity at T_c. As we discussed in the previous section, we should in principle evaluate the matrix elements in terms of QCD variables only in order for performing the Wilsonian matching, which is as yet unavailable from model-independent QCD calculations. Therefore, we make an estimation by extending the dilute gas approximation adopted in the QCD sum-rule analysis in the low-temperature region to the critical temperature with including all the light degrees of freedom expected in the VM. In

[#5]Note that this definition is equivalent to the one used in Ref. [9] at one-loop order.

the HLS/VM theory, both the longitudinal and transverse ρ mesons become massless at the critical temperature since the HLS gauge coupling constant g vanishes. At the critical point, the longitudinal ρ meson which becomes the NG boson σ couples to the vector current whereas the transverse ρ mesons decouple from the theory because of the vanishing g. Thus we assume that thermal fluctuations of the system are dominated near T_c not only by the pions but also by the longitudinal ρ mesons. In evaluating the thermal matrix elements of the non-scalar operators in the OPE, we extend the thermal pion gas approximation employed in Ref. [14] to the longitudinal ρ mesons that figure in our approach. This is feasible since at the critical temperature, we expect the equality $A_4^\rho(T_c) = A_4^\pi(T_c)$ to hold as the massless ρ meson is the chiral partner of the pion in the VM [#6]. It should be noted that, although we use the dilute gas approximation, the treatment here is already beyond the low-temperature approximation adopted in Eq. (23) because the contribution from ρ meson is negligible in the low-temperature region. Since we treat the pion as a massless particle in the present analysis, it is reasonable to take $A_4^\pi(T) \simeq A_4^\pi(T=0)$. We therefore use

$$A_4^\rho(T) \simeq A_4^\pi(T) \simeq A_4^\pi(T=0) \quad \text{for} \quad T \simeq T_c. \tag{35}$$

The properties of the scalar operators giving rise to the condensates are fairly well understood at chiral restoration. We know that the quark condensate must be zero at the critical temperature. Furthermore the value of the gluon condensate at the phase transition is known from lattice calculations to be roughly half of the one in the free space [13]. We therefore can use in what follows the following values at $T = T_c$:

$$\langle \bar{q}q \rangle_T = 0, \quad \langle \frac{\alpha_s}{\pi} G^2 \rangle_T \sim 0.006 \text{GeV}^4 \ . \tag{36}$$

Including the contributions from both pions and massless ρ mesons, Eq. (27) can be expressed as

$$G_A^{(\text{OPE})L(1)} = \frac{32}{105}\pi^4 \frac{T^6}{Q^8}\left[A_4^{\pi(u+d)} + A_4^{\rho(u+d)}\right]. \tag{37}$$

Therefore from Eq. (32), we obtain the deviation δ_{bare} as

$$\delta_{\text{bare}} = 1 - V_{\pi,\text{bare}}^2 = \frac{1}{G_0}\frac{32}{105}\pi^4 \frac{T^6}{\Lambda_M^6}\left[A_4^{\pi(u+d)} + A_4^{\rho(u+d)}\right]. \tag{38}$$

This is the matching condition to be used for determining the value of the bare pion velocity near the critical temperature.

To make a rough estimate of δ_{bare}, we use $A_4^{\pi(u+d)}(\mu = 1 \text{ GeV}) = 0.255$ [14]. This value is arrived at by following Appendix B of [14]. $A_n^{\pi(q)}$ is defined by

$$\langle \pi | \bar{q}\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n} q(\mu) | \pi \rangle = (-i)^{n-1}(p_{\mu_1} \ldots p_{\mu_n} - \text{traces}) A_n^{\pi(q)}(\mu) \ , \tag{39}$$

where

$$A_n^{\pi(q)}(\mu) = 2\int_0^1 dx x^{n-1}[q(x,\mu) + (-1)^n \bar{q}(x,\mu)] \ . \tag{40}$$

[#6] We observe from Refs. [19, 20] that $A_4^{\pi(u+d)}(\mu = 2.4 \text{ GeV}) \sim A_4^{\rho(u+d)}(\mu = 2.4 \text{ GeV})$ even at zero temperature.

For any charge state of the pion (π^0, π^+, π^-), $A_n^{\pi(u+d)}$ $(n = 2, 4)$ can be written in terms of the nth moment of valence quark distribution $V_n^\pi(\mu)$ and sea quark distribution $S_n^\pi(\mu)$ [14],

$$A_n^{\pi(u+d)}(\mu) = 4V_n^\pi(\mu) + 8S_n^\pi(\mu)\ , \tag{41}$$

where

$$\begin{aligned} V_n^\pi &= \int_0^1 dx x^{n-1} v^\pi(x,\mu), \\ S_n^\pi &= \int_0^1 dx x^{n-1} s^\pi(x,\mu). \end{aligned} \tag{42}$$

Simple parameterizations of the valence distribution $v^\pi(x, \mu)$ and the sea distribution $s^\pi(x, \mu)$ can be found in Ref. [17]– see Ref. [18] for the updated results – where the parton distributions in the pions are determined through the π-N Drell-Yan and direct photon production processes. With the leading-order parton distribution functions given in [17], we obtain $A_4^{\pi(u+d)} = 0.255$ at $\mu = 1$ GeV [14]. For the purpose of comparison with the lattice QCD result [19], we need to calculate the value at $\mu = 2.4$ GeV; it comes out to be $A_4^{\pi(u+d)} = 0.18$. The value $A_4^{\pi(u+d)} = 0.18$ is slightly bigger than the ~ 0.13 calculated by the lattice QCD [19], while it is a bit smaller than the ~ 0.22 of Ref. [20]. Note that the value ~ 0.18 agrees with the one determined by lattice QCD [19] within quoted errors. Using $A_2^{\pi(u+d)} = 0.972$ and $A_4^{\pi(u+d)} = 0.255$ and for the range of matching scale ($\Lambda_M = 0.8 - 1.1\,\text{GeV}$), that of QCD scale ($\Lambda_{QCD} = 0.30 - 0.45\,\text{GeV}$) and critical temperature ($T_c = 0.15 - 0.20\,\text{GeV}$), we get

$$\delta_{\text{bare}}(T_c) = 0.0061 - 0.29\,, \tag{43}$$

where the Λ_M dependence of $A_{2,4}^{\pi(u+d)}$ is ignored as it is expected to be suppressed by more than $1/\Lambda_M^6$. Thus we find the *bare* pion velocity to be close to the speed of light:

$$V_{\pi,\text{bare}}(T_c) = 0.83 - 0.99\,. \tag{44}$$

Thanks to the non-renormalization theorem [10], i.e., $v_\pi(T_c) = V_{\pi,\text{bare}}(T_c)$, we arrive at the physical pion velocity at chiral restoration:

$$v_\pi(T_c) = 0.83 - 0.99\,. \tag{45}$$

5 Conclusion

In this paper we lifted the assumption made without justification in the previous paper [6] about the ignorable role of Lorentz symmetry breaking in the bare Lagrangian which led us to the conclusion that the pion velocity at the chiral phase transition equals the speed of light. This feat of doing away with the assumption was made possible by the non-renormalization theorem [10] that states that in the HLS/VM theory, the VM protects the pion velocity from quantum as well as hadronic thermal corrections (at least at one-loop level) and hence the *bare* pion velocity obtained by matching to QCD at a matching scale Λ_M remains un-renormalized by corrections at the critical temperature. By using information available from QCD sum rule calculations made in heat bath, we found that the pion velocity at the chiral transition

temperature remains close to the speed of light. This result is drastically different from the result obtained in the standard chiral theory [8] for the two-flavor QCD which predicts that the pion velocity should go to zero at the critical point. The crucial difference between the HLS/VM result and the standard chiral theory result lies in the degrees of freedom that figure at the phase transition: What accounts for the drastic difference in the prediction is the vector mesons becoming massless in the former at the VM fixed point to which the system is driven by temperature.

It is interesting to compare the result obtained here in heat bath to a similar result obtained in dense medium, i.e., $v_\pi \sim 1$, as the system approaches the critical point. In [21], it was found that a dense matter is driven to chiral restoration by the change in the skyrmion background characterized by the expectation value in "sliding vacua" (SVEV in short) of the scalar field that represents the "soft glue" in QCD trace anomaly. It turns out that the SVEV of the scalar field relevant for this process is locked to the quark condensate $\langle \bar{q}q \rangle$ in such a way that the melting of the quark condensate at chiral restoration precisely corresponds to the melting of the "soft glue" associated with part of the QCD trace anomaly. Remarkably the pion velocity is found to approach the speed of light in dense medium at the critical density in a way analogous to what is found in this paper for the same quantity at the critical temperature. This suggests that the vanishing SVEV of the scalar representing the soft glue in dense medium might be playing the role analogous to the VM fixed point in HLS/VM theory wherein the vector meson mass goes to zero. It remains to be seen whether the same result is obtained with HLS/VM in dense medium.

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